

ST. HENRY'S COLLEGE KITOVU  
 MOCK EXAMINATIONS - 2007  
 ADDITIONAL MATHEMATICS  
 PAPER I  
 2 HOURS 30 MINUTES

**INSTRUCTIONS TO CANDIDATES**

- Answer **eight** questions **only**.
- All working must be shown clearly
- State the degree of accuracy at the end of each answer attempted using a calculator or tables; and indicate **cal** for calculator, or **Tab** for mathematics tables.

1. (a) Given that  $Z_1 = -\sqrt{3} - i$  and  $Z_2 = -4 + 4i$ , find the modulus and argument of:
- (i)  $Z_1$ ,                      (ii)  $Z_2$                       (iii)  $\frac{Z_1}{Z_2}$                       (8 marks)
- (b) Solve the equation  $Z^2 - 4Z + 53 = 0$ , expressing the roots in the form  $a + ib$ , where  $a, b, \in \mathbb{R}$ . Verify that the sum of the roots is 4 and their product is 53. (4 marks)
2. (a) Peter deposits £100 at the beginning of every year to centenary bank on an understanding that at the end of fourteen years he can receive back the money he has paid with 2% compound interest. How much money should he receive? (6 marks)
- (b) The sum of  $m$  terms of an arithmetic progression is  $n$  and the sum of  $n$  terms of the same progression is  $m$ . Find the sum of  $m + n$  terms of this progression. (6 marks)
3. (a) Prove that  $\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$  (3 marks)
- (b) Prove that  $\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$ . Hence solve the equation  $4x^3 - 3x - \sqrt{2} = 0$  by using a suitable substitution of the form  $x = k \cos \Theta$ , then find the possible values of  $\Theta$  between  $0^\circ$  and  $180^\circ$ . Give your answers in surd form. (9 marks)
4. (a) Given that the equation  $4x^2 - 9x - 10 = 0$  has roots  $\alpha$  and  $\beta$ , find the equation whose roots are  $\frac{1}{(\alpha - 2)^2}$  and  $\frac{1}{(\beta - 2)^2}$  (6 marks)
- (b) Prove that  $\log_{15} x = \frac{\log_3 x}{1 + \log_3 5}$ . Hence given that  $\log_3 5 = 1.465$ , find without using tables or calculator  $\log_{15} 27$  correct to 3 significant figures. (6 marks)

5. The locus of P is such that the distance OP is half the distance PR, where O is the origin and R is the point (-3, 6)
- show that the locus of P describes a circle in the  $x - y$  plane.
  - Determine the radius and centre of the circle.
  - Where does it cut the line  $x = 3$ ? (12 marks)
6. (a) Prove by induction that  $ap + ap^2 + ap^3 + \dots + ap^n = \frac{a(1-p^{n+1})}{1-p}$  for all integral values of n. (6 marks)
- (b) Expand  $(1 + 2x)^{1/2}$  in ascending powers of  $x$  as far as the term in  $x^3$ . By substituting  $x = \frac{1}{25}$ , find an approximate value of  $\sqrt{3}$  giving your answer correct to 4 decimal places. (6 marks)
7. (a) Find  $n$  if  ${}^n P_4 = 30 {}^n C_5$  (4 marks)
- (b) A book club offers a choice of 20 books of which a member chooses six. Find the number of different ways in which a member may make his choice  
Given that 12 of the 20 books on offer are novels and that the other 8 are biographies, find the number of different ways in which a member chooses 6 so that
- He has 3 novels and 3 biographies
  - He has at least 4 biographies. (8 marks)
8. (a) Differentiate  $y = \sin x$  from first principles (4 marks)
- (b) If  $y = \frac{\sin x}{x^2}$ , find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  and prove that  $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + (x^2 + 2)y = 0$  (8 marks)
9. (a) Find the area enclosed by that part of  $y = x^2$  for which  $x$  is positive, the  $y - axis$ ,  $y = 1$  and  $y = 4$ . (3marks)
- (b) The triangle formed by the three straight lines  $y = 0$ ,  $y = 2x$  and  $y = 3 - x$  is rotated about the side  $y = 0$ . Find the volume of the solid so generated, and its centre of gravity. (9 marks)
10. (a) Given the points A (3,2) and B(10, 5) determine the coordinates of T which divides the line AB in a ratio 4: -1 (3 marks)

- (b) In a triangle OAB,  $\overrightarrow{OA} = a$ , and  $\overrightarrow{OB} = b$ . P and T are points on OA and OB respectively, such that OT: TB = 3:1 and OP: OA = 1:4. TA and TB intersect at M.

(i) Show that  $\overrightarrow{OM} = \frac{1}{13}(a+9b)$

- (ii) Determine the ratios TM : MA and MP : PB. (9 marks)

11. (a) Given that in any triangle ABC,  $\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c} \cot\left(\frac{A}{2}\right)$ , solve the triangle With two sides 5 and 7 and the included angle  $45^\circ$ . (5marks)

- (b) Find the area of a triangle ABC with vertices A(1,3) B=(4,-5) and C=(9,-1) (7 marks)

12. (a) Prove that  $\log_x(ab) = \log_x a + \log_x b$  and that  $\log_x\left(\frac{a}{b}\right) = \log_x a - \log_x b$  (6 marks)

- (b) Solve the following simultaneous equations

$$\begin{aligned} x - y + Z &= 0 \\ 2x + y + 3Z &= 2 \\ x - 3y + 2Z &= -3. \end{aligned}$$

**END**